INSTRUCCIONES

Para un adecuado uso de este problemario, nos permitimos recomendar lo siguiente:

a) Estudie la teoría pertinente en forma previa.
b) Ejercite la técnica de aprehender con los casos resueltos.
c) Trate de resolver sin ayuda, los ejercicios propuestos.
d) En caso de discrepancia consulte la solución respectiva.
e) En caso de mantener la discrepancia, recurre a la consulta de algún profesor.
f) Al final, hay una cantidad grande de ejercicios sin especificar técnica alguna. Proceda en forma en forma análoga.
g) El no poder hacer un ejercicio, no es razón para frustrarse. Adelante y éxito.

ABREVIATURAS DE USO FRECUENTE

e :	Base de logaritmos neperianos.
$\ell\eta$:	Logaritmo natural o neperiano.
ℓog :	Logaritmo vulgar o de briggs.
$\mathrm{s}e\mathrm{n}$:	Seno.
$\mathrm{arcs}e\mathrm{n}$:	Arco seno.
$\cos$:	Coseno.
$\arccos$:	Arco coseno.
$\mathrm{arc}\,co\mathrm{s}$:	Arco coseno.
τg :	Tangente.
$\mathrm{arc}\,tg$:	Arco tangente.
$\mathrm{co}\,\tau g$	Cotangente.
$\mathrm{arc\,co}\,tg$	Arco cotangente.
$\sec$:	Secante.
arcsec :	Arco secante.
$\cos ec$:	Cosecante.
arcsec :	Arco cosecante.
exp :	Exponencial.
dx :	Diferencial de x.
$\lvert x\rvert$:	Valor absoluto de x.
m.c.m:	Mínimo común múltiplo.

IDENTIFICACIONES USUALES

$\mathrm{s}e\mathrm{n}^n\, x = (\mathrm{s}e\mathrm{n}\, x)^n$

$\mathrm{s}e\mathrm{n}^{-1}\, x = \mathrm{arcs}e\mathrm{n}\, x$

$\ell\eta^n x = (\ell\eta x)^n$

$\ell og^n x = (\ell og x)^n$

$\ell og x = \ell og\,\lvert x\rvert$

IDENTIDADES ALGEBRAICAS

1. Sean a, b: bases; m, n números naturales.

$a^m a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$

$(ab)^n = a^n b^n$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$

$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$a^{-n} = \dfrac{1}{a^n}$

$a^0 = 1, a \neq 0$

2. Sean a, b ,c: bases; m, n números naturales

$(a\pm b)^2 = a^2 + 2ab + b^2$

$(a\pm b)^3 = a^3 \pm 3a^2b + 3ab^2 + b^3$

$(a\pm b)^4 = a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4$

$a^2 - b^2 = (a+b)(a-b)$

$a^{2n} - b^{2n} = (a^n + b^n)(a^n - b^n)$

$a^3 \pm b^3 = (a\pm b)(a^2 \mp ab \pm b^2)$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+ac+bc)$

3. Sean b, n, x, y, z: números naturales

$\ell og(xyz) = \ell og_b x + \ell og_b y + \ell og_b z$

$\ell og_b \left(\frac{x}{y}\right) = \ell og_b x - \ell og_b y$

$\ell og_b x^n = n\ell og_b x$

$\ell og_b \sqrt[n]{x} = \frac{1}{n}\ell og_b x$

$\ell og_b 1 = 0$

$\ell og_b b = 1$

$\ell\eta e = 1$

$\ell\eta \exp x = x = x$

$\ell\eta e^x = x$

$e^{\ell\eta x} = x$

$\exp(\ell\eta x) = x$

IDENTIDADES TRIGONOMETRICAS

1.

$\mathrm{sen} = \frac{1}{\cos ec\theta}$

$\cos\theta = \frac{1}{\mathrm{s}ec\theta}$

$\tau g\theta = \frac{\mathrm{sen}\,\theta}{\cos\theta}$

$\tau g\theta = \frac{1}{\mathrm{co}\,\tau g\theta}$

$\mathrm{sen}^2\theta + \cos^2\theta = 1$

$1 + \tau \mathrm{g}^2\theta = \sec^2\theta$

$1 + \mathrm{co}\,\tau\mathrm{g}^2\theta = \cos ec^2\theta$

$\cos\theta \cos ec\theta = \mathrm{co}\,\tau g\theta$

$\cos\theta\tau g\theta = \mathrm{sen}\,\theta$

2.

(a)

$\mathrm{sen}(\alpha+\beta) = \mathrm{sen}\,\alpha\cos\beta + \cos\alpha\,\mathrm{sen}\,\beta$

$\mathrm{sen}\,2\alpha = 2\,\mathrm{sen}\,\alpha\cos\alpha$

$\mathrm{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\mathrm{sen}^2\alpha = \frac{1-\cos 2\alpha}{2}$

$\mathrm{sen}(\alpha-\beta) = \mathrm{sen}\,\alpha\cos\beta - \cos\alpha\,\mathrm{sen}\,\beta$

(b)

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta$

$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\cos^2\alpha=\frac{1+\cos2\alpha}{2}$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta$

$\cos2\alpha=\cos^2\alpha-\operatorname{sen}^2\alpha=1-2\operatorname{sen}^2\alpha=2\cos^2\alpha-1$

(c)

$\tau g(\alpha+\beta)=\frac{\tau g\alpha+\tau g\beta}{1-\tau g\alpha\tau g\beta}$

$\tau g2\alpha=\frac{2\tau g\alpha}{1-\tau g^2\alpha}$

$\tau g^2\alpha=\frac{1-\cos2\alpha}{1+\cos2\alpha}$

$\tau g(\alpha-\beta)=\frac{\tau g\alpha-\tau g\beta}{1+\tau g\alpha\tau g\beta}$

$\tau g\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{\operatorname{sen}\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\operatorname{sen}\alpha}$

(d)

$\operatorname{sen}\alpha\cos\beta=\frac{1}{2}\left[\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)\right]$

$\cos\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\operatorname{sen}(\alpha+\beta)-\operatorname{sen}(\alpha-\beta)\right]$

$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$

$\operatorname{sen}\alpha\operatorname{sen}\beta=-\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]$

$\operatorname{sen}\alpha+\operatorname{sen}\beta=2\operatorname{sen}\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\operatorname{sen}\alpha-\operatorname{sen}\beta=2\cos\frac{\alpha+\beta}{2}\operatorname{sen}\frac{\alpha-\beta}{2}$

$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha-\cos\beta=-2\operatorname{sen}\frac{\alpha+\beta}{2}\operatorname{sen}\frac{\alpha-\beta}{2}$

(e)

$\operatorname{arcsen}(\operatorname{sen}x)=x$

$\arccos(\cos x)=x$

$\operatorname{arc}\tau g(\tau gx)=x$

$\operatorname{arcco}\tau g(\operatorname{co}\tau gx)=x$

$\operatorname{arcsec}(\sec x)=x$

$\operatorname{arccosec}(\operatorname{cosec}x)=x$

FORMULAS FUNDAMENTALES

Diferenciales

1.- $du = \frac{du}{u}dx$

2.- $d(au) = adu$

3.- $d(u+v) = du + dv$

4.- $d(u^n) = nu^{n-1}du$

5.- $d(\ell\eta u) = \frac{du}{u}$

6.- $d(e^u) = e^u du$

7.- $d(a^u) = a^u \ell\eta a du$

8.- $d(\mathrm{s}e\mathrm{n}\,u) = \cos u du$

9.- $d(\cos u) = -\mathrm{s}e\mathrm{n}\,u du$

10.- $d(\tau gu) = \sec^2 u du$

11.- $d(\mathrm{co}\,\tau gu) = -\mathrm{cosec}^2 u du$

12.- $d(\sec u) = \sec u \tau gu du$

13.- $d(\mathrm{co}\sec u) = -\mathrm{co}\sec u\, \mathrm{co}\,\tau gu du$

14.- $d(\mathrm{arcs}e\mathrm{n}\,u) = \frac{du}{\sqrt{1-u^2}}$

15.- $d(\mathrm{arc}\cos u) = \frac{-du}{\sqrt{1-u^2}}$

16.- $d(\mathrm{arc}\,\tau gu) = \frac{du}{1+u^2}$

17.- $d(\mathrm{arc}\,\mathrm{co}\,\tau gu) = \frac{-du}{1+u^2}$

18.- $d(\mathrm{arc}\sec u) = \frac{du}{u\sqrt{u^2-1}}$

19.- $d(\mathrm{arc}\,\mathrm{co}\sec u) = \frac{-du}{u\sqrt{u^2-1}}$

Integrales

1.- $\int du = u + c$

2.- $\int adu = a\int du$

3.- $\int (du + dv) = \int du + \int dv$

4.- $\int u^n du = \frac{u^{n+1}}{n+1} + c (n \neq -1)$

5.- $\int \frac{du}{u} = \ell\eta |u| + c$

6.- $\int e^u du = e^u + c$

7.- $\int a^u du = \frac{a^u}{\ell\eta a} + c$

8.- $\int \cos u du = \mathrm{s}e\mathrm{n}\,u + c$

9.- $\int \mathrm{s}e\mathrm{n}\,u du = -\cos u + c$

10.- $\int \sec^2 u du = \tau gu + c$

11.- $\int \mathrm{cosec}^2 u du = -\mathrm{co}\,\tau gu + c$

12.- $\int \sec u \tau gu du = \sec u + c$

13.- $\int \mathrm{co}\sec u\, \mathrm{co}\,\tau gu du = -\mathrm{co}\sec u + c$

14.- $\int \frac{du}{\sqrt{1-u^2}} = \mathrm{arcs}e\mathrm{n}\,u + c$

15.- $\int \frac{du}{\sqrt{1-u^2}} = -\mathrm{arc}\cos u + c$

16.- $\int \frac{du}{1+u^2} = \mathrm{arc}\,\tau gu + c$

17.- $\int \frac{du}{1+u^2} = -\mathrm{arc}\,\mathrm{co}\,\tau gu + c$

18.- $\int \frac{du}{u\sqrt{u^2-1}} = \begin{cases} \mathrm{arc}\sec u + c; u > 0 \\ -\mathrm{arc}\sec u + c; u < 0 \end{cases}$

19.- $\int \frac{-du}{u\sqrt{u^2-1}} = \begin{cases} -\mathrm{arc}\,\mathrm{co}\sec u + c; u > 0 \\ \mathrm{arc}\,\mathrm{co}\sec u + c; u < 0 \end{cases}$

OTRAS INTEGRALES INMEDIATAS

1.- $\int \tau g u du = \begin{cases} \ell\eta\left|\sec u\right| + c \\ -\ell\eta\left|\cos u\right| + c \end{cases}$

2.- $\int \operatorname{co}\tau g u du = \ell\eta\left|\operatorname{sen} u\right| + c$

3.- $\int \sec u du = \begin{cases} \ell\eta\left|\sec u + \tau g u\right| + c \\ \ell\eta\left|\tau g u\left(\dfrac{u}{2} + \dfrac{\pi}{4}\right)\right| + c \end{cases}$

4.- $\int \operatorname{cosec} u du = \ell\eta\left|\operatorname{cosec} u - \operatorname{co}\tau g u\right| + c$

5.- $\int \operatorname{sen} h u du = \cos\hbar u + c$

6.- $\int \cos\hbar u du = \operatorname{sen} h u + c$

7.- $\int \tau g h u du = \ell\eta\left|\cos\hbar u\right| + c$

8.- $\int \operatorname{co}\tau g h u du = \ell\eta\left|\operatorname{sen}\hbar u\right| + c$

9.- $\int \sec h u du = \operatorname{arc}\tau g h(\operatorname{sen} h u) + c$

10.- $\int \operatorname{cosec} h u du = -\operatorname{arc}\operatorname{co}\tau g h(\cos h u) + c$

11.- $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \begin{cases} \operatorname{arcsen}\dfrac{u}{a} + c \\ -\operatorname{arcsen}\dfrac{u}{a} + c \end{cases}$

12.- $\int \dfrac{du}{\sqrt{u^2 \pm a^2}} = \ell\eta\left|u + \sqrt{u^2 \pm a^2}\right| + c$

13.- $\int \dfrac{du}{u^2 + a^2} = \begin{cases} \dfrac{1}{a}\operatorname{arc}\tau g\dfrac{u}{a} + c \\ \dfrac{1}{a}\operatorname{arc}\operatorname{co}\tau g\dfrac{u}{a} + c \end{cases}$

14.- $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ell\eta\left|\dfrac{u-a}{u+a}\right| + c$

15.- $\int \dfrac{du}{u\sqrt{a^2 \pm u^2}} = \dfrac{1}{a}\ell\eta\left|\dfrac{u}{a + \sqrt{a^2 \pm u^2}}\right| + c$

16.- $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \begin{cases} \dfrac{1}{a}\operatorname{arc}\cos\dfrac{u}{a} + c \\ \dfrac{1}{a}\operatorname{arc}\sec\dfrac{u}{a} + c \end{cases}$

17.- $\sqrt{u^2 \pm a^2}du = \dfrac{u}{2}\sqrt{u^2 \pm a^2} \pm \dfrac{a^2}{2}\ell\eta\left|u + \sqrt{u^2 \pm a^2}\right| + c$

18.- $\int \sqrt{a^2 - u^2}du = \dfrac{u}{2}\sqrt{a^2 - u^2} + \dfrac{a^2}{2}\operatorname{arcsen}\dfrac{u}{a} + c$

19.- $\int e^{au}\operatorname{sen} bu du = \dfrac{e^{au}(a\operatorname{sen} bu - b\cos bu)}{a^2 + b^2} + c$

20.- $\int e^{au}\cos bu du = \dfrac{e^{au}(a\cos bu + b\operatorname{sen} bu)}{a^2 + b^2} + c$

Realmente, algunas de estas integrales no son estrictamente inmediatas; tal como se verá mas adelante y donde se desarrollan varias de ellas.

CAPITULO 1

INTEGRALES ELEMENTALES

El Propósito de este capitulo, antes de conocer y practicar las técnicas propiamente tales; es familiarizarse con aquellas integrales para las cuales basta una transformación algebraica elemental.

EJERCICIOS DESARROLLADOS

1.1.- Encontrar: $\int e^{\ell\eta x^2} x dx$

Solución.- Se sabe que: $e^{\ell\eta x^2} = x^2$

Por lo tanto: $\int e^{\ell\eta x^2} x dx = \int x^2 x dx = \int x^3 dx = \frac{x^4}{4} + c$

Respuesta: $\int e^{\ell\eta x^2} x dx = \frac{x^4}{4} + c$, Fórmula utilizada: $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$

1.2 .- Encontrar: $\int 3a^7 x^6 dx$

Solución.-

$$\int 3a^7 x^6 dx = 3a^7 \int x^6 dx = 3a^7 \frac{x^7}{7} + c$$

Respuesta: $\int 3a^7 x^6 dx = 3a^7 \frac{x^7}{7} + c$, Fórmula utilizada: del ejercicio anterior.

1.3.- Encontrar: $\int (3x^2 + 2x + 1) dx$

Solución.-

$$\int (3x^2 + 2x + 1) dx = \int (3x^2 + 2x + 1) dx = \int 3x^2 dx + \int 2x dx + \int dx$$

$$= 3\int x^2 dx + 2\int x dx + \int dx = \not{3} \frac{x^3}{\not{3}} + \not{2} \frac{x^2}{\not{2}} + x + c = x^3 + x^2 + x + c$$

Respuesta: $\int (3x^2 + 2x + 1) dx = x^3 + x^2 + x + c$

1.4.- Encontrar: $\int x(x+a)(x+b) dx$

Solución.-

$$\int x(x+a)(x+b) dx = \int x\left[x^2 + (a+b)x + ab\right] dx = \int \left[x^3 + \left(a+b\right)x^2 + abx\right] dx$$

$$= \int x^3 dx + \int (a+b) x^2 dx + \int abx dx = \int x^3 dx + (a+b)\int x^2 dx + ab\int x dx$$

$$= \frac{x^4}{4} + (a+b)\frac{x^3}{3} + ab\frac{x^2}{2} + c$$

Respuesta: $\int x(x+a)(x+b)dx = \frac{x^4}{4} + \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$

1.5.- Encontrar: $\int (a+bx^3)^2\, dx$

Solución.-

$$\int (a+bx^3)^2 dx = \int (a^2 + 2abx^3 + b^2x^6)dx = \int a^2 dx + \int 2abx^3 dx + \int b^2x^6 dx$$

$$= a^2\int dx + 2ab\int x^3 dx + b^2\int x^6 dx = a^2x + 2ab\frac{x^4}{4} + b^2\frac{x^7}{7} + c$$

Respuesta: $\int (a+bx^3)^2 dx = a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7} + c$

1.6.- Encontrar: $\int \sqrt{2px}dx$

Solución.-

$$\int \sqrt{2px}dx = \int \sqrt{2p}x^{\frac{1}{2}}dx = \sqrt{2p}\int x^{1/2}dx = \sqrt{2p}\frac{x^{1/2}}{2/3} + c = \frac{2\sqrt{2p}x^{2/3}}{3} + c$$

Respuesta: $\int \sqrt{2px}dx = \frac{2\sqrt{2p}x\sqrt{x}}{3} + c$

1.7.-Encontrar: $\int \frac{dx}{\sqrt[n]{x}}$

Solución.-

$$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-1/n}dx = \frac{x^{\frac{-1}{n}+1}}{\frac{-1}{n}+1} + c = \frac{x^{\frac{-1+n}{n}}}{\frac{-1+n}{n}} + c = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$$

Respuesta: $\int \frac{dx}{\sqrt[n]{x}} = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$

1.8.- Encontrar: $\int (nx)^{\frac{1-n}{n}} dx$

Solución.-

$$\int (nx)^{\frac{1-n}{n}} dx = \int n^{\frac{1-n}{n}} x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1}{n}-1} dx$$

$$= = n^{\frac{1-n}{n}} \frac{x^{\frac{1}{n}-1+1}}{\frac{1}{n}-1+1} + c = n^{\frac{1-n}{n}} \frac{x^{\frac{1}{n}}}{\frac{1}{n}} + c = n^{\frac{1-n}{n}} nx^{\frac{1}{n}} + c = n^{\frac{1-n}{n}+1} x^{\frac{1}{n}} + c = n^{\frac{1-n+n}{n}} x^{\frac{1}{n}} + c = n^{1/n}x^{1/n} + c$$

Respuesta: $\int (nx)^{\frac{1-n}{n}} dx = \sqrt[n]{nx} + c$

1.9.- Encontrar: $\int (a^{2/3} - x^{2/3})^3 dx$

Solución.-

$$\int (a^{2/3} - x^{2/3})^3 dx = \int \left[\left(a^{2/3}\right)^3 - 3\left(a^{2/2}\right)^2 x^{2/3} + 3a^{2/3}\left(x^{2/3}\right)^2 - \left(x^{2/3}\right)^3 \right] dx$$

$$=\int(a^2-3a^{4/3}x^{2/3}+3a^{2/3}x^{4/3}-x^2)dx=\int a^2dx-\int 3a^{4/3}x^{2/3}dx+\int 3a^{2/3}x^{4/3}dx-\int x^2dx$$

$$=a^2\int dx-3a^{4/3}\int x^{2/3}dx+3a^{2/3}\int x^{4/3}dx-\int x^2dx=a^2x-3a^{4/3}\frac{x^{5/3}}{5/3}+3a^{2/3}\frac{x^{7/3}}{7/3}-\frac{x^3}{3}+c$$

$$=a^2x-\frac{9a^{4/3}x^{5/3}}{5}+\frac{9a^{2/3}x^{7/3}}{7}-\frac{x^3}{3}+c$$

Respuesta: $\int(a^{2/3}-x^{2/3})^3dx=a^2x-\frac{9a^{4/3}x^{5/3}}{5}+\frac{9a^{2/3}x^{7/3}}{7}-\frac{x^3}{3}+c$

1.10.- Encontrar: $\int(\sqrt{x}+1)(x-\sqrt{x}+1)dx$

Solución.-

$$\int(\sqrt{x}+1)(x-\sqrt{x}+1)dx=(x\sqrt{x}-\cancel{(\sqrt{x})^2}+\cancel{\sqrt{x}}+\cancel{x}-\cancel{\sqrt{x}}+1)dx$$

$$=\int(x\sqrt{x}+1)dx=\int(xx^{1/2}+1)dx=\int(x^{3/2}+1)dx=\int x^{3/2}dx+\int dx=\frac{x^{5/2}}{5/2}+x+c=\frac{2x^{5/2}}{5}+x+c$$

Respuesta: $\int(\sqrt{x}+1)(x-\sqrt{x}+1)dx=\frac{2x^{5/2}}{5}+x+c$

1.11.- Encontrar: $\int\frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}$

Solución.-

$$\int\frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}=\int\frac{(x^4-x^2-2)dx}{x^{2/3}}=\int\frac{x^4}{x^{2/3}}dx-\int\frac{x^2}{x^{2/3}}dx-\int\frac{2}{x^{2/3}}dx$$

$$=\int x^{10/3}dx-\int x^{4/3}dx-2\int x^{-2/3}dx=\frac{x^{\frac{10}{3}+1}}{\frac{10}{3}+1}-\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}-2\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}=\frac{x^{13/3}}{\frac{13}{3}}-\frac{x^{7/3}}{\frac{7}{3}}-2\frac{x^{1/3}}{1/3}+c$$

$$=3\frac{x^{13/3}}{13}-3\frac{x^{7/3}}{7}-6x^{1/3}+c=3\frac{\sqrt[3]{x^{13}}}{13}-3\frac{\sqrt[3]{x^7}}{7}-6\sqrt[3]{x}+c=3\frac{x^4\sqrt[3]{x}}{13}-3\frac{x^2\sqrt[3]{x}}{7}-6\sqrt[3]{x}+c$$

Respuesta: $\int\frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}=\left(\frac{3x^4}{13}-\frac{3x^2}{7}-6\right)\sqrt[3]{x}+c$

1.12.- Encontrar: $\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx$

Solución.-

$$\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx=\int\frac{(x^{2m}-2x^mx^n+x^{2n})}{\sqrt{x}}dx=\int\frac{(x^{2m}-2x^mx^n+x^{2n})}{x^{1/2}}dx$$

$$=\int(x^{2m-1/2}-2x^{m+n-1/2}+x^{2n-1/2})dx=\frac{x^{2m-1/2+1}}{2m-1/2+1}-\frac{2x^{m+n+1/2}}{m+n+1/2}+\frac{x^{2n+1/2}}{2n+1/2}+c$$

$$=\frac{x^{\frac{4m+1}{2}}}{\frac{4m+1}{2}}-\frac{2x^{\frac{2m+2n+1}{2}}}{\frac{2m+2n+1}{2}}+\frac{x^{\frac{4n+1}{2}}}{\frac{4n+1}{2}}+c=\frac{2x^{\frac{4m+1}{2}}}{4m+1}-\frac{4x^{\frac{2m+2n+1}{2}}}{2m+2n+1}+\frac{2x^{\frac{4n+1}{2}}}{4n+1}+c$$

$$= \frac{2x^{2m}\sqrt{x}}{4m+1} - \frac{4x^{m+n}\sqrt{x}}{2m+2n+1} + \frac{2x^{2n}\sqrt{x}}{4n+1} + c$$

Respuesta: $\int \frac{(x^m - x^n)^2}{\sqrt{x}} dx = \sqrt{x}\left(\frac{2x^{2m}}{4m+1} - \frac{4x^{m+n}}{2m+2n+1} + \frac{2x^{2n}}{4n+1}\right) + c$

1.13.- Encontrar: $\int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx$

Solución.-

$$\int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx = \int \frac{a^2 - 4a\sqrt{ax} + 6xa - 4x\sqrt{ax} + x^2}{\sqrt{ax}} dx$$

$$= \int \frac{a^2}{(ax)^{1/2}} dx - \int \frac{4a\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{6ax}{(ax)^{1/2}} dx - \int \frac{4x\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{x^2}{(ax)^{1/2}} dx$$

$$= \int a^2 a^{-1/2} x^{-1/2} dx - \int 4a dx + \int 6a a^{-1/2} x x^{-1/2} dx - \int 4x dx + \int a^{-1/2} x^2 x^{-1/2} dx$$

$$= a^{3/2} \int x^{-1/2} dx - 4a \int dx + 6a^{1/2} \int x^{1/2} dx - 4\int x dx + a^{-1/2} \int x^{3/2} dx$$

$$= a^{3/2} \frac{x^{-1/2+1}}{\frac{-1}{2}+1} - 4ax + 6a^{1/2} \frac{x^{1/2+1}}{\frac{1}{2}+1} - 4\frac{x^{1+1}}{1+1} + a^{-1/2} \frac{x^{3/2+1}}{\frac{3}{2}+1} + c$$

$$= a^{3/2} \frac{x^{1/2}}{\frac{1}{2}} - 4ax + 6a^{1/2} \frac{x^{3/2}}{\frac{3}{2}} - 4\frac{x^2}{2} + a^{-1/2} \frac{x^{5/2}}{\frac{5}{2}} + c$$

$$= 2a^{3/2} x^{1/2} - 4ax + 4a^{1/2} x^{3/2} - 2x^2 + 2a^{-1/2} \frac{x^{5/2}}{5} + c$$

Respuesta: $\int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx = 2a^{3/2} x^{1/2} - 4ax + 4a^{1/2} x^{3/2} - 2x^2 + \frac{2x^3}{5\sqrt{xa}} + c$

1.14.- Encontrar: $\int \frac{dx}{x^2 - 10}$

Solución.-

Sea: $a = \sqrt{10}$, Luego: $\int \frac{dx}{x^2 - 10} = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ell\eta \left|\frac{x-a}{x+a}\right| + c$

$$= \frac{1}{2\sqrt{10}} \ell\eta \left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right| + c = \frac{\sqrt{10}}{20} \ell\eta \left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right| + c$$

Respuesta: $\int \frac{dx}{x^2 - 10} = \frac{\sqrt{10}}{20} \ell\eta \left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right| + c$

1.15.- Encontrar: $\int \frac{dx}{x^2 + 7}$

Solución.- Sea: a=$\sqrt{7}$, Luego: $\int \frac{dx}{x^2 + 7} = \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arc}\tau g \frac{x}{a} + c$

$$\frac{1}{\sqrt{7}}\operatorname{arc}\tau g\frac{x}{\sqrt{7}}+c=\frac{\sqrt{7}}{7}\operatorname{arc}\tau g\frac{\sqrt{7}x}{a}+c$$

Respuesta: $\int\frac{dx}{x^2+7}=\frac{\sqrt{7}}{7}\operatorname{arc}\tau g\frac{\sqrt{7}x}{a}+c$

1.16.- Encontrar: $\int\sqrt{\frac{dx}{4+x^2}}$

Solución.-

Sea: $a=2$, Luego: $\int\frac{dx}{\sqrt{4+x^2}}=\int\frac{dx}{\sqrt{a^2+x^2}}=\ell\eta\left|x+\sqrt{a^2+x^2}\right|+c$

$$=\ell\eta\left|x+\sqrt{4+x^2}\right|+c$$

Respuesta: $\int\frac{dx}{\sqrt{4+x^2}}=\ell\eta\left|x+\sqrt{4+x^2}\right|+c$

1.17.- Encontrar: $\int\frac{dx}{\sqrt{8-x^2}}$

Solución.-

Sea: $a=\sqrt{8}$, Luego: $\int\frac{dx}{\sqrt{8-x^2}}=\int\frac{dx}{\sqrt{a^2-x^2}}=\operatorname{arcs}e\operatorname{n}\frac{x}{a}+c$

$$=\operatorname{arcs}e\operatorname{n}\frac{x}{\sqrt{8}}+c=\operatorname{arcs}e\operatorname{n}\frac{x}{2\sqrt{2}}+c$$

Respuesta: $\int\frac{dx}{\sqrt{8-x^2}}=\operatorname{arcs}e\operatorname{n}\frac{\sqrt{2}x}{4}+c$

1.18.- Encontrar: $\int\frac{dy}{x^2+9}$

Solución.-

La expresión: $\frac{1}{x^2+9}$ actúa como constante, luego:

$$\int\frac{dy}{x^2+9}=\frac{1}{x^2+9}\int dy=\frac{1}{x^2+9}y+c=\frac{y}{x^2+9}+c$$

Respuesta: $\int\frac{dy}{x^2+9}=\frac{y}{x^2+9}+c$

1.19.- Encontrar: $\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx$

Solución.-

$$\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx=\int\sqrt{\frac{2+x^2}{4-x^4}}dx-\int\sqrt{\frac{2-x^2}{4-x^4}}dx$$

$$=\int\sqrt{\frac{\cancel{2+x^2}}{(2-x^2)\cancel{(2+x^2)}}}dx-\int\sqrt{\frac{\cancel{2-x^2}}{\cancel{(2-x^2)}(2+x^2)}}dx=\int\frac{dx}{\sqrt{2-x^2}}-\int\frac{dx}{\sqrt{2+x^2}}$$

Sea: $a=\sqrt{2}$, Luego: $\int \frac{dx}{\sqrt{a^2-x^2}}-\int \frac{dx}{\sqrt{a^2+x^2}}=\operatorname{arcsen}\frac{x}{a}-\ell\eta\left|x+\sqrt{a^2+x^2}\right|+c$

$$=\operatorname{arcsen}\frac{x}{\sqrt{2}}-\ell\eta\left|x+\sqrt{(\sqrt{2})^2+x^2}\right|+c=\operatorname{arcsen}\frac{x}{\sqrt{2}}-\ell\eta\left|x+\sqrt{2+x^2}\right|+c$$

Respuesta: $\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}dx=\operatorname{arcsen}\frac{x}{\sqrt{2}}-\ell\eta\left|x+\sqrt{2+x^2}\right|+c$

1.20.- Encontrar: $\int \tau g^2 x dx$

Solución.-

$$\int \tau g^2 x dx=\int(\sec^2 x-1)dx=\int \sec^2 x dx-\int dx=\tau g x-x+c$$

Respuesta: $\int \tau g^2 x dx=\tau g x-x+c$

1.21.- Encontrar: $\int \operatorname{co}\tau g^2 x dx$

Solución.-

$$\int \operatorname{co}\tau g^2 x dx=\int(\operatorname{cos}ec^2 x-1)dx=\int \operatorname{cos}ec^2 x dx-\int dx=-\operatorname{co}\tau g x-x+c$$

Respuesta: $\int \operatorname{co}\tau g^2 x dx=-\operatorname{co}\tau g x-x+c$

1.22.- Encontrar: $\int \frac{dx}{2x^2+4}$

Solución.-

$$\int \frac{dx}{2x^2+4}=\int \frac{dx}{2(x^2+2)}=\frac{1}{2}\int \frac{dx}{x^2+2}=\frac{1}{2}\frac{1}{\sqrt{2}}\operatorname{arc}\tau g\frac{x}{\sqrt{2}}+c\quad=\frac{\sqrt{2}}{4}\operatorname{arc}\tau g\frac{\sqrt{2}x}{2}+c$$

Respuesta: $\int \frac{dx}{2x^2+4}=\frac{\sqrt{2}}{4}\operatorname{arc}\tau g\frac{\sqrt{2}x}{2}+c$

1.23.- Encontrar: $\int \frac{dx}{7x^2-8}$

Solución.-

$$\int \frac{dx}{7x^2-8}=\int \frac{dx}{7(x^2-\frac{8}{7})}=\int \frac{dx}{7\left[(x^2-(\sqrt{\frac{8}{7}})^2\right]}=\frac{1}{7}\int \frac{dx}{\left[x^2-(\sqrt{\frac{8}{7}})^2\right]}$$

$$=\frac{1}{7}\frac{1}{2(\sqrt{\frac{8}{7}})}\ell\eta\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right|+c=\frac{1}{14\frac{\sqrt{8}}{\sqrt{7}}}\ell\eta\left|\frac{x-\sqrt{\frac{8}{7}}}{x+\sqrt{\frac{8}{7}}}\right|+c=\frac{\sqrt{7}}{14\sqrt{8}}\ell\eta\left|\frac{\sqrt{7}x-\sqrt{8}}{\sqrt{7}x+\sqrt{8}}\right|+c$$

$$=\frac{1}{4\sqrt{14}}\ell\eta\left|\frac{\sqrt{7}x-2\sqrt{2}}{\sqrt{7}x+2\sqrt{2}}\right|+c=\frac{\sqrt{14}}{56}\ell\eta\left|\frac{\sqrt{7}x-2\sqrt{2}}{\sqrt{7}x+2\sqrt{2}}\right|+c$$

Respuesta: $\int \frac{dx}{7x^2-8}=\frac{\sqrt{14}}{56}\ell\eta\left|\frac{\sqrt{7}x-2\sqrt{2}}{\sqrt{7}x+2\sqrt{2}}\right|+c$

1.24.- Encontrar: $\int \frac{x^2dx}{x^2+3}$

Solución.-

$$\int \frac{x^2dx}{x^2+3} = \int (1-\frac{3}{x^2+3})dx = \int dx - 3\int \frac{dx}{x^2+3} = \int dx - 3\int \frac{dx}{x^2+(\sqrt{3})^2}$$

$$= x - 3\frac{1}{\sqrt{3}}\operatorname{arc}\tau g\,\frac{x}{\sqrt{3}} + c == x - \sqrt{3}\operatorname{arc}\tau g\,\frac{\sqrt{3}x}{3} + c$$

Respuesta: $\int \frac{x^2dx}{x^2+3} = x - \sqrt{3}\operatorname{arc}\tau g\,\frac{\sqrt{3}x}{3} + c$

1.25.- Encontrar: $\int \frac{dx}{\sqrt{7+8x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7+8x^2}} = \int \frac{dx}{\sqrt{(\sqrt{8}x)^2+(\sqrt{7})^2}} = \frac{1}{\sqrt{8}}\ell\eta\left|\sqrt{8}x+\sqrt{7+8x^2}\right| + c$$

Respuesta: $\int \frac{dx}{\sqrt{7+8x^2}} = \frac{\sqrt{2}}{4}\ell\eta\left|\sqrt{8}x+\sqrt{7+8x^2}\right| + c$

1.26.- Encontrar: $\int \frac{dx}{\sqrt{7-5x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{7})^2-(\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}}\operatorname{arcs}e\operatorname{n} x\frac{\sqrt{5}}{\sqrt{7}} + c$$

Respuesta: $\int \frac{dx}{\sqrt{7-5x^2}} = \frac{\sqrt{5}}{5}\operatorname{arcs}e\operatorname{n}\frac{\sqrt{35}x}{7} + c$

1.27.- Encontrar: $\int \frac{(a^x-b^x)^2dx}{a^xb^x}$

Solución.-

$$\int \frac{(a^x-b^x)^2dx}{a^xb^x} = \int \frac{(a^{2x}-2a^xb^x+b^{2x})}{a^xb^x}dx = \int \frac{a^{2x}}{a^xb^x}dx - \int \frac{2\cancel{a^xb^x}}{\cancel{a^xb^x}}dx + \int \frac{b^{\cancel{2}x}}{a^x\cancel{b^x}}dx$$

$$= \int \frac{a^x}{b^x}dx - \int 2dx + \int \frac{b^x}{a^x}dx = \int\left(\frac{a}{b}\right)^x dx - 2\int dx + \int\left(\frac{b}{a}\right)^x dx = \frac{(a/b)^x}{\ell\eta\frac{a}{b}} - 2x + \frac{(b/a)^x}{\ell\eta\frac{b}{a}} + c$$

$$= \frac{(a/b)^x}{\ell\eta a-\ell\eta b} - 2x + \frac{(b/a)^x}{\ell\eta b-\ell\eta a} + c = \frac{(a/b)^x}{\ell\eta a-\ell\eta b} - 2x - \frac{(b/a)^x}{\ell\eta a-\ell\eta b} + c$$

$$= \frac{\left(\frac{a^x}{b^x}-\frac{b^x}{a^x}\right)}{\ell\eta a-\ell\eta b} - 2x + c$$

Respuesta: $\int \frac{(a^x-b^x)^2dx}{a^xb^x} = \frac{\left(\frac{a^{2x}-b^{2x}}{a^xb^x}\right)}{\ell\eta a-\ell\eta b} - 2x + c$

1.28.- Encontrar: $\int sen^2 \frac{x}{2} dx$

Solución.-

$$\int sen^2 \frac{x}{2} dx = \int \frac{1-\cos \not{2}\frac{x}{\not{2}}}{2} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos x dx$$

$$= \frac{x}{2} - \frac{senx}{2} + c$$

Respuesta: $\int sen^2 \frac{x}{2} dx = \frac{x}{2} - \frac{senx}{2} + c$

1.29.- Encontrar: $\int \frac{dx}{(a+b)+(a-b)x^2}; (0 < b < a)$

Solución.-

Sea: $c^2 = a+b,\ \ d^2 = a-b,$; luego $\int \frac{dx}{(a+b)+(a-b)x^2} = \int \frac{dx}{c^2+d^2x^2}$

$$\int \frac{dx}{d^2\left(\frac{c^2}{d^2}+x^2\right)} = \frac{1}{d^2}\int \frac{dx}{\left(\frac{c}{d}\right)^2 + x^2} = \frac{1}{d^{\not{2}}}\frac{1}{\frac{c}{\not{d}}} arctg \frac{x}{c/d} + c = \frac{1}{cd} arctg \frac{dx}{c} + c$$

$$= \frac{1}{\sqrt{a+b}\sqrt{a-b}} arctg \frac{\sqrt{a-b}x}{\sqrt{a+b}} + c = \frac{1}{\sqrt{a^2-b^2}} arctg \sqrt{\frac{a-b}{a+b}} x + c$$

Respuesta: $\int \frac{dx}{(a+b)+(a-b)x^2} = \frac{1}{\sqrt{a^2-b^2}} arctg \sqrt{\frac{a-b}{a+b}} x + c$

1.30.-Encontrar: $\int \frac{dx}{(a+b)-(a-b)x^2}; (0 < b < a)$

Solución.-

Sea: $c^2 = a+b,\ d^2 = a-b,$ Luego: $\int \frac{dx}{(a+b)-(a-b)x^2} = \int \frac{dx}{c^2-d^2x^2}$

$$= \int \frac{dx}{d^2\left(\frac{c^2}{d^2}-x^2\right)} = \frac{1}{d^2}\int \frac{dx}{\left(\frac{c}{d}\right)^2 - x^2} = -\frac{1}{d^{\not{2}}}\frac{1}{\frac{2c}{\not{d}}} \ell\eta \left|\frac{x-c/d}{x+c/d}\right| + c = -\frac{1}{2cd} \ell\eta \left|\frac{dx-c}{dx+c}\right| + c$$

$$= -\frac{1}{2\sqrt{a^2-b^2}} \ell\eta \left|\frac{\sqrt{a-b}x - \sqrt{a+b}}{\sqrt{a-b}x + \sqrt{a+b}}\right| + c$$

Respuesta: $\int \frac{dx}{(a+b)-(a-b)x^2} = -\frac{1}{2\sqrt{a^2-b^2}} \ell\eta \left|\frac{\sqrt{a-b}x - \sqrt{a+b}}{\sqrt{a-b}x + \sqrt{a+b}}\right| + c$

1.31.- Encontrar: $\int \left[\left(a^{2x}\right)^0 - 1\right] dx$

Solución.-